The Application of Simulated Moving Bed Chromatography for the Separation Between 2,6- and 2,7-Dimethylnaphthalene

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Abstract—The possibility of applying SMB chromatography for the separation of 2,6-DMN and 2,7-DMN was investigated by numerical simulation based on the single column chromatography experimental data. It was found that 2,6-DMN could be separated from 2,7-DMN in the ODS-modified silica gel by using methanol and water as the mobile phase. A systematic method of obtaining model parameters for the SMB simulation from single column chromatography experiments was presented. The adsorption isotherms of 2,6-DMN and 2,7-DMN were obtained by the pulse method. The mass transfer kinetics was very fast, indicating that the mobile phase and the stationary phase are very close to equilibrium for both 2,6-DMN and 2,7-DMN. SMB simulation was performed by using the model parameters. Pure 2,6-DMN and 2,7-DMN can be obtained by selecting suitable operating conditions, indicating that SMB chromatography can be employed to separate 2,6-DMN and 2,7-DMN. The influence of operating parameters (feed flow rates, raffinate flow rate, and switch time) was also investigated to recommend the optimum operating conditions.

Key words: SMB, Chromatography, Dimethylnaphthalene, Separation

INTRODUCTION

2,6-Dimethylnaphthalene (DMN) is of great practical interest as a feed stock for the high performance polyester, polyethylenenaphthalate (PEN), which provides superior strength and heat resistance compared to polyethylene terephthalate (PET) in applications such as film, containers, mold parts, etc. [Chem Systems, 1993]. Sources of DMNs are the alkylation of naphthalene with methanol, the transalkylation of polymethylaromatics, or the aromatic petroleum fraction [Hagen and Hung, 1992; Olah and Olah, 1976]. But it is very difficult to recover pure 2,6-DMN from DMN isomers, especially to separate 2,6-DMN from 2,7-DMN, due to their similar physical properties [Rota et al., 1996]. For the separation of 2,6-DMN extraction or selective crystallization, which are not effective in the separation between 2,6-DMN and 2,7-DMN, has been used. Therefore, it is necessary to develop an effective and economical separation method for the separation between 2,6-DMN and 2,7-DMN.

SMB (simulated moving bed) chromatography has been attracting considerable attention in the last few years for the preparative and production scale separation of fine chemicals, especially enantiomers [Wu et al., 1999; Pedeferri et al., 1999]. In the SMB chromatography, the countercurrent flow is simulated by repeatedly switching the ports of the inlet (feed, solvent) and outlet (extract, raffinate) streams one column in the direction of the liquid flow. Continuous operation in SMB chromatography eliminates drawbacks of batch chromatography, dilution of components and low adsorbent efficiency. It allows the separation of components with separation factors near unity, providing high resolution, yield, and purity [Ruthven and Ching, 1989; Zhong and Guiochon, 1997].

SMB chromatography can be a promising alternative separation

technique for the economical separation of pure 2,6-DMN and 2,7-DMN, whose separation is nearly impossible and uneconomical in other separation techniques (e.g., distillation and crystallization) due to their very close physical properties. However, the operating conditions of SMB chromatography cannot be obtained simply but can be obtained by rigorous experiments and/or simulations. Therefore, a systematic SMB simulation method would be a great help to guide selecting the optimum operating conditions for the separation between 2,6-DMN and 2,7-DMN in SMB chromatography application.

In this study the possibility of applying SMB chromatography for the separation of 2,6-DMN and 2,7-DMN was investigated. The SMB operating conditions for 2,6-DMN complete separation, along with the influences of operation parameters on the 2,6-DMN separation (purity, yield, and recovery), were investigated in ODS (octadecylsilyl) modified silica gel. Systematic methods for model parameter estimation of SMB chromatography from the single column chromatography data were presented and the influence of operating parameters (feed flow rate, raffinate flow rate, switch time, etc) was investigated by the numerical simulation of SMB chromatography.

EXPERIMENTS

Preliminary HPLC experiments were performed to find the most suitable stationary phase and mobile phase composition for 2,6-DMN separation. Stationary phases such as ODS modified silica gel and β-cyclodextrin modified silica gel were investigated. 2,6-DMN was not separated from 2,7-DMN in the stationary phase of β-cyclodextrin modified silica gel, although it had been reported that 2,6-DMN could be separated from its isomer by the formation of inclusion complex between 2,6-DMN and β-cyclodextrin [Sybilska et al., 1993]. It seems that 2,6-DMN and 2,7-DMN have sim-

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ilar affinity for the formation of inclusion complexes. Using ODS-modified silica gel as the stationary phase, it was found that 2,6-DMN could be separated from 2,7-DMN, and several experiments were performed to find the best suitable mobile phase compositions.

The HPLC consisted of standard components: high pressure delivery pump (Younglin, Korea; flow range 0-10 ml/min), six-port sample injection valve with 5 µl sampling loop (Rheodyne), column, and continuous flow UV detector (Varian, model 2550). The detector signal was transmitted into a PC for further data processing. HPLC column (stainless steel, 250×4.6 mm ID) was slurry packed with J'sphere ODS-H80 (YMC, mean diameter=4 µm). The column porosity, ε=0.6498, was determined by injecting uracil as an inert substance. Reagent grade 2,6-DMN and 2,7-DMN (Wako) were used. The mobile phases employed (methanol, acetonitrile, ethanol) were of chromatographic grade and water was distilled and deionized. HPLC experiments were carried out at 25 °C.

RESULTS AND DISCUSSION

1. Chromatographic Separation Between 2,6-DMN and 2,7-DMN

Among the employed mobile phases, combinations of methanol and water showed the most desirable separation between 2,6-DMN and 2,7-DMN. The retention time measurements were carried out by using several mobile phase compositions of water and methanol in ODS-modified silica gel. The retention factors as a function of water content in mobile phase are presented in Fig. 1. Retention factors increase with water content in the mobile phase. The separation selectivity (retention factor difference between 2,6-DMN and 2,7-DMN) increases rapidly to 25% of water content and then slowly with water content. However, the increase in water content has some harmful effects on the chromatography operation. First, viscosity of the mobile phase increases drastically with the increase in water content due to the increasing intermolecular hydrogen bonding between water and methanol [Guichon et al., 1994], increasing the operating pressure of the column as shown in Fig. 1. Also, the
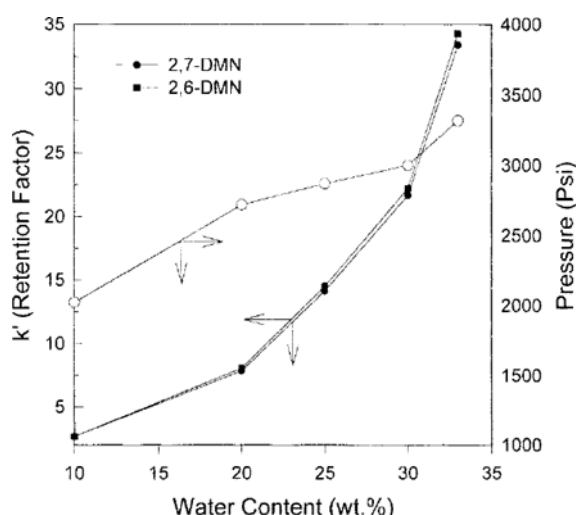

Fig. 1. Influence of water content in the mobile phase on the retention factors of 2,6-DMN and 2,7-DMN and the operating pressure.

Table 1. The influence of mobile phase flow rates on the 2,6- and 2,7-DMN retention factors [mobile phase: methanol and water (75 : 25)]

Mobile phase flow rate (ml/min)	Retention factor		Retention factor difference
	2,6-DMN	2,7-DMN	
0.6	26.28	25.63	0.65
0.7	22.39	21.83	0.56
0.8	19.63	19.14	0.49
1.0	14.51	14.10	0.41
1.1	12.71	12.82	0.21

Table 2. The influence of column temperature on the 2,6- and 2,7-DMN retention factors [mobile phase: methanol and water (75 : 25); mobile phase flow rate: 1 ml/min]

Column temperature (°C)	Retention factor		Retention factor difference
	2,6-DMN	2,7-DMN	
25	14, 51	14.10	0.41
30	10.15	10.14	0.01
40	7.80	7.80	0.0

solubility of DMNs in the mobile phase decreases with water content, limiting the capacity of chromatography. Considering separation selectivity, operating pressure, and DMN solubility, it seems reasonable to use water content of 25% in the mobile phase.

The effect of mobile phase flow rate on the retention factors was investigated and is summarized in Table 1. It seems that a mobile phase flow rate of 1 ml/min was suitable for the separation. The effect of temperature on retention factors was also investigated and presented in Table 2. It was observed that retention factors decrease with the increase in temperature. The separation selectivity also decreases with temperature increases and no separation between 2,6-DMN and 2,7-DMN was observed above 30 °C.

2. Adsorption Isotherms of 2,6-DMN and 2,7-DMN

Adsorption isotherms of 2,6-DMN and 2,7-DMN were obtained by the pulse method [Choi et al., 2000; Guichon et al., 1994; Row and Larin, 1995]. A steady stream of a solution of 2,6-DMN (or 2,7-DMN) in the mobile phase (25% water in methanol) was pumped through the ODS-modified silica gel column until equilibrium was reached, i.e., until the breakthrough of the constant concentration plateau had been reached. Then a 5 µl pulse of higher concentration sample of 2,6-DMN (or 2,7-DMN) was injected and the retention time was measured. Repeating the experiments many times while increasing the concentration of 2,6-DMN (or 2,7-DMN) in the mobile phase, we obtained data of retention time as a function of the concentration of 2,6-DMN (or 2,7-DMN) in the mobile phase (Fig. 2).

The retention time of the pulse is related to the equilibrium isotherm through the following equation:

$$t_R(C) = t_o\left(1 + \frac{1-\varepsilon}{\varepsilon}\frac{dq}{dC}\right) \quad (1)$$

where $t_R(C)$ is the retention time, C is the solute concentration in the mobile phase, q is the solute concentration in the stationary phase, t_o is the mobile phase hold up time, and dq/dC is the slope of the isotherm. Eq. (1) gives the pulse retention time, and permits the de-
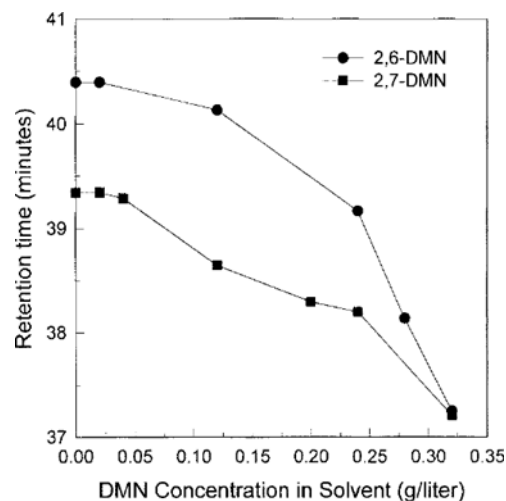
Fig. 2. Influence of DMN (2,6-DMN or 2,7-DMN) concentration in the mobile phase on the DMN (2,6-DMN or 2,7-DMN, respectively) retention time (mobile phase: methanol and water (75 : 25), mobile phase flow rate: 1 ml/min).

rivation of the slope of the isotherm at the plateau concentration. The Langmuir isotherm was determined by repeating the procedure a number of times while progressively increasing the concentration of 2,6- or 2,7-DMN, as described previously, and by numerical integration of the plot of dq/dC vs. C obtained.

The adsorption isotherms of 2,6-DMN and 2,7-DMN were obtained by using Eq. (1) and the data in Fig. 2. Fig. 3 represents 2,6-DMN and 2,7-DMN isotherms at the temperature of 25 °C. Both isotherms are linear, yielding a constant separation factor of 1.05. The Langmuir type isotherm equation was found to fit both 2,6-DMN and 2,7-DMN isotherms. The resulting adsorption isotherm of 2,6-DMN is

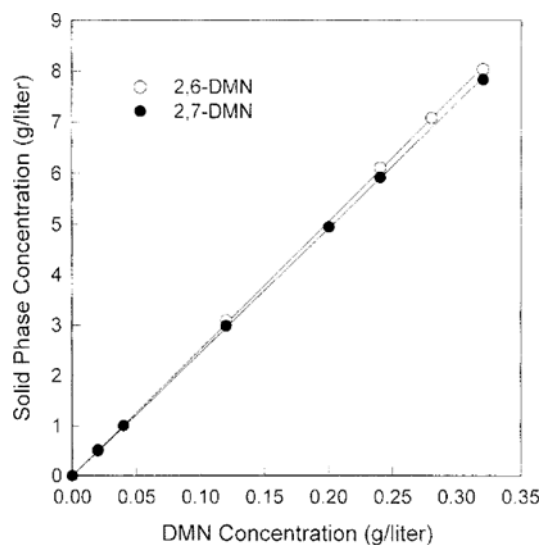$$q_{2,6}^* = \frac{26.3C_{2,6}}{1+146.5C_{2,6}} \tag{2}$$

Fig. 3. Equilibrium adsorption isotherms of 2,6-DMN and 2,7-DMN at 25 °C.

and that of 2,7-DMN is

$$q_{2,7}^* = \frac{25.15C_{2,7}}{1+83.79C_{2,7}}, \tag{3}$$

where the subscript indicates each component and the dimension of q and C is g/ml.

The binary Langmuir isotherms can be easily extended to the competitive Langmuir isotherms due to the basic Langmuir isotherm assumption that there is no interaction between molecules on different sites [Guichon et al., 1994]. The binary system adsorption isotherms of 2,6-DMN and 2,7-DMN were obtained by extending the single component Langmuir isotherms of 2,6-DMN and 2,7-DMN to the competitive Langmuir isotherm model. The adsorption isotherm of 2,6-DMN in binary system is

$$q_{2,6}^* = \frac{26.3C_{2,6}}{1+146.5C_{2,6}+83.79C_{2,7}} \tag{4}$$

and that of 2,7-DMN is

$$q_{2,7}^* = \frac{25.15C_{2,7}}{1+146.5C_{2,6}+83.79C_{2,7}}. \tag{5}$$

3. Parameter Estimation for SMB Chromatography

A differential mass balance for component i in the bulk mobile phase around a tiny control volume of the column, neglecting radial dispersion, gives

$$\frac{\partial C_i}{\partial t} = -u\frac{\partial C_i}{\partial z} + D_{ax}\frac{\partial^2 C_i}{\partial z^2} - \frac{1-\varepsilon}{\varepsilon}\frac{\partial C_{s,i}}{\partial t} \tag{6}$$

where u is the mobile phase velocity and D_{ax} is the axial dispersion coefficient. If the overall mass transfer resistance is assumed to be dominant in the liquid film around the particle, another mass balance in the particulate phase gives

$$\frac{\partial C_{s,i}}{\partial t} = k_{eff,i}(q_i^* - q_i) \tag{7}$$

where

$$q_i^* = f_i(C_i, C_{i\neq i}). \tag{8}$$

As shown in Eqs. (6)-(8), some model parameters are needed to solve the SMB chromatography equations. The parameters are the porosity ε, the axial dispersion coefficient D_{ax}, adsorption isotherms, and the effective mass transfer coefficients $k_{eff,i}$.

The porosity of the column was determined by the injection of an inert substance (i.e., uracil). The axial dispersion coefficient can be estimated from the following relation of the height equivalent to a theoretical plate (HETP) if no adsorption is taking place.

$$\frac{D_{ax}}{u} = \frac{HETP}{2} = \frac{L}{2N} = \frac{\sigma^2}{2t_R^2} \tag{9}$$

The HETP of the ODS column was found to be 8.03×10^{-4} cm. The estimated axial dispersion coefficient for the column is 7.455×10^{-5} cm²/sec when the mobile phase flow rate is 1 ml/min.

Since the adsorption isotherms were estimated by using the pulse method and the pulse method does not use the elution profiles for other estimations, the elution profiles contain information about the isotherm, axial dispersion, and mass transfer resistance. As long as the axial dispersion coefficient and adsorption isotherms are esti-

mated as described above, the overall mass transfer coefficients could be estimated by fitting simulated single column model [using Eqs. (6)-(8)] to experimental elution profiles of 2,6-DMN and 2,7-DMN. The estimated overall mass transfer coefficients of 2,6-DMN and 2,7-DMN were 75 s^{-1} and 85 s^{-1}, respectively.

If the mass transfer kinetics between and across the mobile phase and stationary phase in the column is very fast (k_{eff}>50 s^{-1}), these phases are very close to equilibrium [Guichon et al., 1994]. Therefore, it can be assumed that the mass transfer kinetics is very fast and the two phases are in equilibrium for 2,6-DMN and 2,7-DMN. Then, the following equilibrium relation can be used instead of Eqs. (7) and (8),

$$C_{s,i} = q_i^* = \Gamma_i(C_{i}, C_{j\neq i}) \tag{10}$$

Since there is no need to estimate the overall mass transfer coefficient from the elution profiles, the axial dispersion coefficient is re-estimated by fitting simulated single column model to the elution profiles of 2,6-DMN and 2,7-DMN. The fitting result gives the axial dispersion coefficient of 7.46×10^{-5} cm^2/sec. which is almost same as that estimated from the HETP.

The simulated elution profile of the 2,6-DMN and 2,7-DMN mixture obtained by using the model parameters is presented in Fig. 4 together with the experimental elution profile for comparison. The experimental elution profile was obtained by injecting 5μl sample of 1 : 1 mixture of 2,6-DMN and 2,7-DMN. The concentration of 2,6-DMN and 2,7-DMN in sample is 0.5 g/l in the solution of methanol-water (75 : 25) for each component. The simulated single column model can predict well the experimental elution profile, indicating the appropriateness of the estimated model parameters and the binary system adsorption isotherms.

4. SMB Chromatography Simulation

In SMB chromatography, the column is divided into a number of element columns, and countercurrent contact is achieved by advancing the solvent, extract, feed, and raffinate ports simultaneously at fixed switch time intervals in the direction of the mobile phase
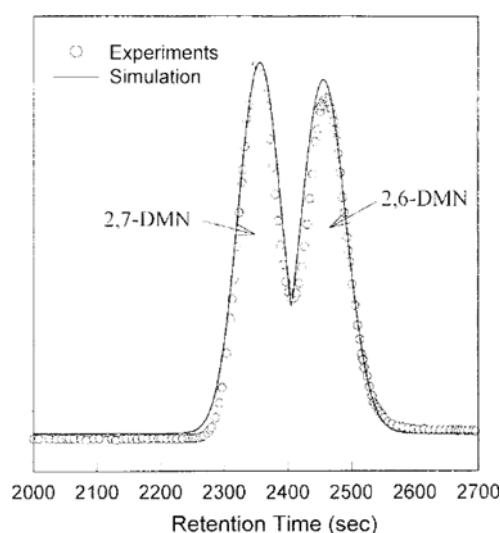

Fig. 4. Experimental and simulated elution profile of 2,6-DMN and 2,7-DMN (mobile phase: methanol and water (75 : 25), mobile phase flow rate: 1 ml/min, injection sample: 1 : 1 mixture of 2,6-DMN and 2,7-DMN, injection volume: 5 μl).
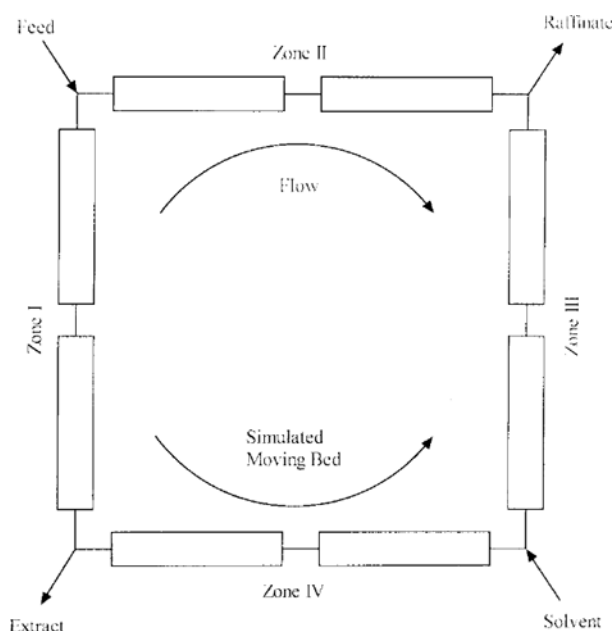
Fig. 5. Schematic diagram of eight-element column SMB chromatography.

flow [Beste et al., 2000]. Here we consider a typical eight-element column system with two element columns in each zone (Fig. 5). The differential mass balance in zone j (j=I, II, III, IV) for component i (i=1, 2) is

$$\frac{\partial C_{i,j}}{\partial t} = -u_j \frac{\partial C_{i,j}}{\partial z} + D_{ax} \frac{\partial^2 C_{i,j}}{\partial z^2} - \frac{1-\varepsilon}{\varepsilon} \frac{\partial C_{s,i,j}}{\partial t} \tag{11}$$

and the equilibrium model of Eq. (10) was used since the DMN mass transfer kinetics is very fast (k_{eff}>50 s^{-1}). The competitive Langmuir isotherms of 2,6-DMN and 2,7-DMN [Eqs. (4) and (5)] were used to account for the equilibrium between the mobile and stationary phases.

The initial and boundary conditions of SMB chromatography are

$$C_{i,j}(z,0) = 0, \quad q_{i,j}(z,0) = 0 \tag{12a}$$

$$C_{i,j}^{in} = C_{i,j}|_{z=0} - \frac{D_{ax}}{u_j} \frac{\partial C_{i,j}}{\partial z}\Big|_{z=0} \tag{12b}$$

$$\frac{\partial C_{i,j}}{\partial z}\Big|_{z=L} = 0. \tag{12c}$$

For numerical simulation the partial differential equations of the SMB model were converted into a set of ordinary differential equations (ODE) by the method of lines. The ODE set was then solved with initial and boundary conditions by the adaptive step size Runge-Kutta technique [Press et al., 1992]. The error tolerance of the ODE solver in the FORTRAN code was 10^{-8}.

The internal diameter (D) and length of an element column (L) were chosen as 1.2 cm and 20 cm, respectively. Other model parameters were those estimated above (i.e., ε=0.6498; D$_{ax}$=7.46×10^{-5} cm^2/sec). The feed concentration was 0.5 g/l for each component. A simulation result showing complete separation between 2,6-DMN and 2,7-DMN is presented in Fig. 6, which shows axial concentra-
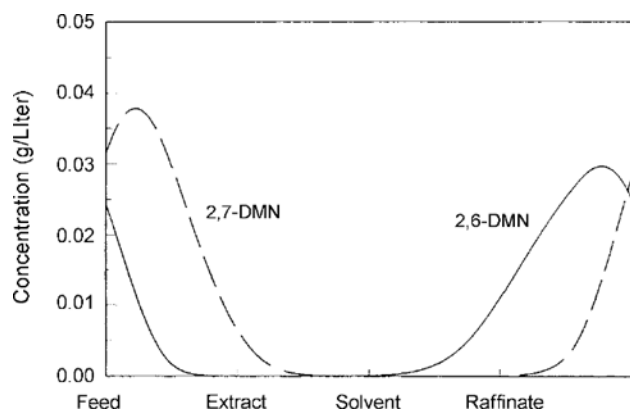
Fig. 6. Axial concentration profiles of 2,6-DMN and 2,7-DMN in SMB chromatography.

Fig. 7. Regions of the (m_2, m_3) plane with different separation regimes. Dotted boundaries identify the square triangle for a linear system.

tion profiles of 2,6-DMN and 2,7-DMN. The extract is pure 2,7-DMN and the raffinate is pure 2,6-DMN.

A starting set of operating parameters (i.e., feed, extract, raffinate, solvent, internal recycle flow rates, and switch time) was estimated by the triangle method [Mazzoti et al., 1997]. The conditions of a complete separation between 2,6-DMN and 2,7-DMN were investigated by carrying out simulations under various sets of operating parameters. To describe the conditions of a complete separation, the parameter of m_j, which is directly related with the SMB operating variables [mobile phase flow rate and switch time (t_{sw})] in the four zones of the SMB chromatography, was defined as

$$m_j = \frac{\varepsilon}{(1-\varepsilon)} \frac{(u_j - L/t_{sw})}{L/t_{sw}},$$
(13)

In the four-zone SMB chromatography, m_j of zone II and III plays the important role in the separation performance [Mazzotti et al., 1997]. The m_2-m_3 plot was obtained keeping the recycle flow rate (i.e., the inlet mobile phase flow rate into the first element column in section IV) and the switch time constant. The total inlet and outlet flow rates were kept constant for the simulation as 3.0 ml/min, and the switch time was set as 455 sec. The recycle flow rate was 30 ml/min.

Fig. 7 shows the m_2-m_3 plot where six regions are defined: a region of complete separation, two regions where only one outlet stream is pure, and three regions where there is no separation. The dotted triangle in Fig. 7 represents the boundary of the complete separation region predicted by the triangle method based on the linear isotherm. It is notable that the simulated complete separation region is much smaller and curved than that predicted by the triangle method.

The shrinkage of the complete separation region arises from the nonlinearity of an SMB system such as nonlinear adsorption isotherms (Langmuir isotherms), axial dispersion effect, and high feed concentration. The size of the complete separation region would increase with the decrease in the feed concentrations and eventually increase to the dotted triangle at very low feed concentrations.

The specific mobile phase consumption and productivity per unit mass of stationary phase improve while moving away from the diagonal in the m_2-m_3 plot, and the optimum value will be on the vertex of the complete separation region. However, SMB operation with the operating parameters at the vertex is unstable since the vertex
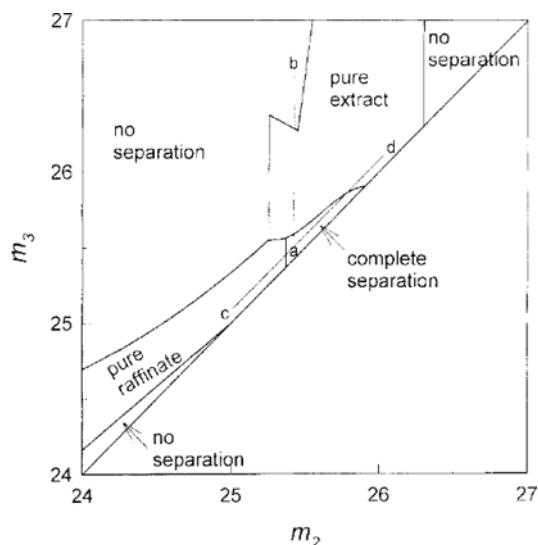
is the boundary of four regions of separation. Even small disturbances during operation would lead to deviation from the complete separation region. Therefore, the optimum operating conditions will be the points just below the vertex.

The effects of feed and raffinate flow rates (or switch time) on the SMB performance were investigated. The line a-b in Fig. 7 shows the effect of increases in feed flow rate, showing that increasing feed flow rate makes the operating conditions cross pure extract, starting from complete separation region and ending up in no separation region. The line c-d in Fig. 7 indicates the effect of increases in raffinate flow rate (or switch time), showing that increasing raffinate flow rate (or switch time) makes the operation conditions cross the complete separation region, starting from pure raffinate and ending up in pure extract. This information can be used to plan the experimental investigation to find the optimum operating conditions.

CONCLUSIONS

2,6-DMN is of great practical interest as a feedstock for the high performance polyester, PEN. However, it is very difficult to separate 2,6-DMN and 2,7-DMN due to their very close physical properties. In this study, the possibility of applying SMB chromatography, a continuous chromatography, for the separation between 2,6-DMN and 2,7-DMN was investigated by numerical simulation of SMB chromatography and single column chromatography experiments. For this purpose, a systematic method of obtaining model parameters for SMB simulation from single chromatography experiments was presented.

It was found that 2,6-DMN could be separated from 2,7-DMN in the ODS-modified silica gel by using methanol-water as the mobile phase, and the suitable mobile phase composition would be 25% water in methanol. The adsorption isotherms of 2,6-DMN and 2,7-DMN were obtained by the pulse method. Model parameters were estimated from the elution profiles by fitting a simulated single column model. The single column simulation using the estimated mod-

el parameters can predict successfully the experimental elution profiles of 2,6-DMN and 2,7-DMN in the ODS column, indicating the appropriateness of the estimated model parameters.

SMB simulation was performed by using the estimated model parameters. It was found that pure 2,6-DMN and 2,7-DMN could be obtained by selecting suitable operating conditions, suggesting that SMB chromatography can be employed to separate 2,6-DMN and 2,7-DMN. The influence of operating parameters (feed flow rates, raffinate flow rate, and switch time) was also investigated to suggest the optimum operating conditions.

The systematic model parameter estimation and SMB simulation method would be a great help to guide the selection of operating conditions in experimental or industrial SMB chromatography for the separation between 2,6-DMN and 2,7-DMN, since the selection of the optimum operating condition is very difficult and time consuming.

ACKNOWLEDGEMENT

NOMENCLATURE

C : solute concentration in the mobile phase [g/ml]
C_s : solute concentration in the stationary phase [g/ml]
D_{ax} : axial dispersion coefficient [cm^2/s]
$k_{\it eff}$: mass transfer coefficient [1/s]
L : column length [cm]
N : column plate number
q : solute concentration in the stationary phase [g/cm]
u : mobile phase velocity [cm/s]
t_o : mobile phase hold up time [s]
t_R : residence time [s]
t_{sw} : column switching time [s]
z : axial position as measured from the inlet [cm]

Greek Letters

ε : column porosity
σ : standard deviation

Subscripts

2,6   : 2,6-DMN
2,7   : 2,7-DMN
i   : component
j   : separation zone

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